Manuscript for trapping analysis

**I Introduction**

The manipulation of particles via electromagnetic (EM) waves is a powerful and versatile technique, primarily employed for two key objectives: particle acceleration and trapping. In the context of acceleration, one prominent mechanism is autoresonance, wherein a charged particle sustains phase synchronization with an EM wave through nonlinear effects. A well-known example is the Gyro-Resonant Accelerator (Gyrac), in which electrons resonate within a spatially varying magnetic field while interacting with a fixed-frequency EM wave (see The Gyrac: A Proposed Gyro-Resonant Accelerator of Electrons). Additional acceleration mechanisms include betatron resonance (Particle Acceleration in Relativistic Laser Channels) and ponderomotive-force-driven acceleration (Laser Electron Accelerator, T. Tajima).

For particle trapping, spatial confinement can be achieved using methods such as optical tweezers or ponderomotive potential wells generated by the spatial interference patterns of laser beats (High-Efficiency Nonadiabatic Trapping of Electrons in the Ponderomotive Potential Wells of Laser Beats). Velocity-selective trapping, on the other hand, can be realized through Landau resonant, wherein electrons with velocities near the phase velocity of a longitudinal wave become confined within the associated potential well.

Although the interaction between test particles and EM waves has been extensively examined, few studies have incorporated the role of a static electric field in such interactions, despite its relevance in phenomena such as runaway electron generation in tokamaks. Building upon analytical approaches used to study electron pitch-angle scattering in EM waves (e.g., Paper1), we extend existing models by introducing a static electric field, thereby uncovering a previously unexplored form of resonant trapping in EM wave–particle interactions. In this study, we numerically investigate a novel trapping phenomenon in which electrons within a static electric field become confined at the anomalous Doppler resonant velocity under the influence of a left-hand circularly polarized plane EM wave in a uniform magnetic field. Under these conditions, the work performed by the static electric field is continuously converted into gyrokinetic energy, thereby sustaining resonance with the EM wave.

The paper is organized as follows: In Section II, we present a mathematical analysis of the interaction between an electromagnetic wave and an electron in the presence of a static electric field aligned with a uniform background magnetic field. Section III provides a numerical investigation of the resulting trapping phenomenon. In Section IV, we analyze the physical mechanism and derive the critical threshold conditions required for trapping. Finally, Section V offers a discussion of the results and a summary of the main conclusions.

**II electron-electromagnetic wave interaction analysis**

**2.1 Field equations**

To analyze interaction under the anomalous Doppler resonant, we consider an electromagnetic wave with left-hand circular polarization (LCP) whose phase velocity satisfies < c, where ω is the angular frequency and k is the wavenumber. The wavevector k is aligned parallel to the uniform background magnetic field B₀ = B₀ẑ. The wave's magnetic field perturbation B̃ is characterized by the dimensionless parameter κ ≡ |B̃|/B₀, such that the total magnetic field becomes **B** = B₀ẑ + B̃. The system includes a static electric field E₀ = E₀ẑ, with the total electric field given by **E** = E₀ẑ + Ẽ, where Ẽ represents the LCP wave electric field components:

Here, is the wave electric field amplitude, and the polarization satisfies the LCP condition for electron cyclotron interactions in the anomalous Doppler regime.

Faraday’s law requires the associated magnetic field to be

Here .

**2.2 Transformation to the wave frame**

In wave frame, which denotes as prime and moves at constant velocity with respect to the lab frame, the fields are

Where . Substituting the wave fields Eq. , Eq. into Eq., Eq. gives and .

Since and {**x**, ict} are relativistic four-vectors, we have

here is the wavenumber in the wave frame. The wave magnetic field is than

the motion equation of the charge particle in the prime frame is

Where . Note that ,, and differ and should not be confused with each other

Here is the nonrelativistic electron cyclotron frequency in the lab frame with , according to Eq. , Eq. , we have

Combining with Eq. , we obtain

Where

**2.3 Construction of pseudo-potential problem**

Taking the derivative of Eq. (14) with respect to t’ gives

Here we ignore the derivative of as we assume and . the time derivation of wave magnetic field in wave frame is

where .Substitution using Eq. ,Eq. and Eq. into Eq. gives

Ignoring the high order of terms gives

here . Introduce the

The characterizes the **frequency mismatch** relative to the anomalous doppler resonant condition . From the definition of we have:

Since {} are four-vector, we have Substituting Eq. and Eq. into Eq. gives

The actually is nonlinear function of , which is different with no static electric field as in paper  [1],since

To express approximately linearly with , the following condition must be satisfied:

This leads to the requirement:

Since ,we have

Thus, the condition for linearity reduces to . For instance, when we find , which satisfies the inequality. Therefore, in the regime where , the dependence of on can be considered approximately linear.

**2.4 connect with**

To obtain the relationship between and , inserting Eq. and Eq. into the time derivation of gives

where

and

Finally, we have

The equation is first-order linear differential equation with the form:

The solution of z is

here and , . Since P << 1, we have

Ignoring the second term of the Eq. , the expression can be written as:

The substitution of Eq. into Eq. gives

Integrating Eq. with and ignoring the second term on the right sight gives:

Noting that t = 0 corresponds to z’=0, since z = 0 at t= 0, and recalling the four vectors {}, we have the relation:

Since is in the y direction when z’ = 0, it follows that

Where is defined by and , . With these definitions, Eq. becomes

The substitution of Eq. into Eq. gives

Simplifying Eq. and substituting for gives:

The Eq. could also be written as

Where

Multiplying Eq. by and integrating gives a pseudo-energy equation

Where

Here ,, The pseudo potential can only be solved numerically, as it does not have a regular form involving only the parameter , here also depends on t’ , which is related to and .

**2.5 Initial condition**

From Eq. and , we see that

At the start where t = 0 and z = 0, we have

**2.6 Solve**

According to Eq., multiplying on both sides gives

Ignoring third term on the right side and reorganize the equation, we have

According to Eq. , multiplying on both sides gives

Add Eq. and Eq. , we have

Which means energy change ratio equal to work done by static electric field.

Integrating both side with t’ gives

**III. Numerical study of the trapping effect**

According to Eq. and Eq. , the velocity could be numerically solved by ode45, and subsequently, can be determined from Eq. . Then, using the four-vector {}, we have:

Main topics here: 1. Unstrapping situation (the velocity (\beta\_z and \beta\_\perp), the phase of xi and dxidt, the pseudo-potential) 2. Trapping situation 3. Critical threshold.